Finite-time singularities in f(R, T) gravity and the effect of conformal anomaly

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Abstract

We investigate $f(R,T)$ gravity models (R is the curvature scalar and T is the trace of the stress-energy tensor of ordinary matter) that are able to reproduce the four known types of future finite-time singularities. We choose a suitable expression for the Hubble parameter in order to realise the cosmic acceleration and we introduce two parameters, α and H_s, which characterise each type of singularity. We address conformal anomaly and we observe that it cannot remove the sudden singularity or the type IV one, but, for some values of α, the big rip and the type III singularity may be avoided. We also find that, even without taking into account conformal anomaly, the big rip and the type III singularity may be removed thanks to the presence of the T contribution of the $f(R,T)$ theory.

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1 Introduction

Observation indicates that the current expansion of the universe is accelerating [1, 2]. There are two large branches of cosmology attempting to explain this phenomenon [3]-[12]: *i*) Introducing dark energy in the framework of general relativity is the first; *ii*) The other is investigating modified versions of the gravitational theory. In this paper we follow the latter approach. The most widely renown modified theory is $f(R)$-gravity, in which the action is determined by an arbitrary function $f(R)$ of

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the Ricci scalar R [5]-[8]. Another interesting modified theory of gravity is $f(G)$-gravity, where $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet invariant ($R_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ are the Ricci tensor and the Riemann tensor respectively).

In the framework of general relativity, it is well known that a phantom phase usually ends up in the type I (big rip) finite-time future singularity (FTFS) [13, 14]. It has also been demonstrated, e.g. in [15]-[17], that the dark energy equation of state parameter may show all four possible types of FTFS and that there is no qualitative difference between dark energy and modified gravity. Thus, $f(R)$-gravity dark energy models also may bring the universe evolution to all four possible FTFS [18]-[22]. Another interesting aspect of $f(R)$ modified gravity is that it may provide models that remove FTFS by adding, for example, a R^2-term [18, 19, 21, 23]. The $f(G)$-gravity also is a class of modified theories that may lead to finite-time future singularities [19, 24].

In the present paper, we approach $f(R,T)$-gravity, where T is the trace of the energy-momentum tensor of ordinary matter, and investigate models that may lead to the four FTFS. This type of modified theory of gravity has been first developed by Harko *et al.* [25], who derive the gravitational field equations in the metric formalism, as well as the equations of motion of test particles, which follow from the covariant divergence of the stress-energy tensor. They also analysed the Newtonian limit of the equations of motion, and provide a constraint on the magnitude of the extra-acceleration by investigating the perihelion precession of Mercury. In [26], $f(R,T)$ models have been constructed describing the transition from the matter-dominated phase to the late-times accelerated one and in [27] the authors consider cosmological scenarios based on $f(R,T)$ theory and the function $f(R,T)$ is numerically reconstructed from holographic dark energy.

In this paper, we consider the special case in which the function $f(R,T)$ is the usual Einstein-Hilbert term plus a correction $g(T)$, i.e. $f(R,T) = R + 2g(T)$. A suitable expression is assumed for the Hubble parameter, which may provide the four FTFS. For some values of the parameter α, differential equations of $g(T)$ are established and solved. Another aspect which we study are quantum effects due to conformal anomaly near the singularities. We observe that conformal anomaly cannot remove the sudden singularity or the type IV singularity. However, for some values of the parameter α, the big rip and the type III singularity may be avoided. Another important result that we obtain is that, even without taking into account quantum effects, the big rip and the type III singularity may be avoided thanks to the contribution T of the modified theory.

The paper is organized as follows. In Sec. 2, we present the general formulation of the theory addressing the special case $f(R,T) = R + 2g(T)$ and obtaining the models that reproduce each type of FTFS. In Sec. 3, we investigate quantum effects due to conformal anomaly. We present our conclusions and perspectives in Sec. 4.

2 f(R, T)-gravity

2.1 The model

We assume a modification of general relativity in which the Ricci scalar R is replaced by an arbitrary function $f(R,T)$. The total action reads

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(R,T) + \mathcal{L}\right] , \tag{1}$$

where $\mathcal{L}$ is the matter Lagrangian density and $T = g^{\mu\nu}T_{\mu\nu}$ is the trace of the matter energy-momentum tensor $T_{\mu\nu}$ which is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} . \tag{2}$$

Varying the action (1) with respect to the metric, one obtains the gravitational field equations

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right) f_R(R,T) = T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu} , \tag{3}$$

where f_R and f_T denote the derivatives of f with respect to R and T, respectively; ∇_μ is the covariant derivative and $\Theta_{\mu\nu}$ is defined by

$$\Theta_{\mu\nu} \equiv g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} = -2T_{\mu\nu} + g_{\mu\nu}\mathcal{L}_m - 2g^{\alpha\beta}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial^{\alpha\beta}} . \tag{4}$$

We adopt the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$, as the space-time geometry, and we assume the matter content of the universe to be a perfect fluid, whose the stress tensor is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} , \tag{5}$$

where u_μ is the four-velocity and satisfies the condition $u_\mu u^\mu = 1$; ρ and p are, respectively, the energy density and the pressure of ordinary matter. The matter Lagrangian density can be taken as $\mathcal{L}_m = -p$, which implies $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$. Thus, Eq. (3) becomes

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right) f_R(R,T) = T_{\mu\nu} + f_T(R,T)T_{\mu\nu} + pg_{\mu\nu}f_T(R,T) . \tag{6}$$

Our ansatz for the function f is the following: $f(R,T) = R + 2g(T)$, where $g(T)$ is an arbitrary function of T. Then, Eq. (6) becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu} + 2g_T(T)T_{\mu\nu} + \left[2pg_T(T) + g(T)\right]g_{\mu\nu} . \tag{7}$$

The components 00 and ii of Eq. (7) read

$$3H^2 = \rho_{eff} = \rho + 2\left(\rho + p\right)g_T(T) + g(T) , \tag{8}$$

$$-2\dot{H} - 3H^2 = p_{eff} = p - g(T) . \tag{9}$$

Here, ρ_{eff} and p_{eff} are the effective energy density and pressure, respectively. Combining Eqs. (8) and (9), we get the following equations

$$-2\dot{H} = \rho\,(1+\omega)\,[1+2g_T(T)] \ , \tag{10}$$

$$6\frac{\ddot{a}}{a} = 2\,[\rho + g(T)] - \rho\,(1+\omega)\,[2g_T(T)+3] \ , \tag{11}$$

where we have defined $\omega \equiv p/\rho$.

Note that, in order to provide an accelerated expanding universe, the function $g(T)$ has to satisfy the condition

$$2g(T) - 2\rho\,(1+\omega)\,g_T(T) > \rho\,(1+3\omega) \ . \tag{12}$$

2.2 Finite-time singularities in $R + 2g(T)$ gravity

We are interested in the modified $f(R,T)$ gravity models that produce some types of finite-time future singularities (see [28, 29]), given by the Hubble parameter

$$H = h(t_s - t)^{-\alpha} \ , \tag{13}$$

where h and t_s are positive constants and $t < t_s$. This expression is chosen for guaranteeing an expanding universe. Due to fact that H or some of its derivatives (and therefore the curvature) could become singular when t is close to t_s, the parameter α could be either positive or negative. If $\alpha > 0$, then H diverges as the singularity time is approached. The same expression of the Hubble parameter provides the acceleration of the universe, since the strong energy condition is violated. On the other hand, when $\alpha < 0$, a relevant positive constant H_s may be introduced and interpreted as the Hubble parameter at the singularity time, such that $H = h(t_s - t)^{-\alpha} + H_s$. In this case, some derivatives of H (and therefore the curvature) become singular. Note the particular case $\alpha = 0$, which corresponds to the de Sitter space (which we do not treat in this paper).

Depending on the value of α, different expressions can be found for the scale factor.

If $\alpha = 1$, the scale factor reads

$$a(t) = \bar{a}(t_s - t)^{-h} \ , \tag{14}$$

where $\bar{a}$ is an integration constant.

If $\alpha > 0$ and $\alpha \neq 1$, we get

$$a(t) = \bar{a}\exp\left[\frac{h(t_s - t)^{1-\alpha}}{\alpha - 1}\right] \ . \tag{15}$$

Finally, if $\alpha < 0$ and $H_s > 0$, the scale factor behaves as

$$a(t) = \bar{a}\exp\left\{-(t_s - t)\left[H_s - \frac{h(t_s - t)^{-\alpha}}{\alpha - 1}\right]\right\} \ . \tag{16}$$

The finite future time singularities can be classified as follows [30, 31]:

- Type I (big rip): for $t \to t_s$, $a \to \infty$, ρ_{eff} and $|p_{eff}| \to \infty$ at $t = t_s$. This corresponds to $\alpha \geq 1$.
- Type II (sudden): for $t \to t_s$, $a \to a_s$, $\rho_{eff} \to \rho_s$ and $|p_{eff}| \to \infty$. It corresponds to $-1 < \alpha < 0$.
- Type III: for $t \to t_s$, $a \to a_s$, $\rho_{eff} \to \infty$ and $|p_{eff}| \to \infty$. This corresponds to $0 < \alpha < 1$.
- Type IV: for $t \to t_s$, $a \to a_s$, $\rho_{eff} \to 0$, $p_{eff} \to 0$ and higher derivatives of H diverge. This corresponds to the case $\alpha < -1$, but with α not an integer number.

We investigate $f(R,T) = R + 2g(T)$ gravity models which generate the above types of finite-time singularities. This corresponds to determine which form of $g(T)$ gives rise to each type of singularity.

We introduce an arbitrary auxiliary field ϕ which should not be confused with the scalar field on which the matter action depends. Write $g(T)$ as

$$g(T) = P(\phi)T + Q(\phi) \ . \tag{17}$$

By using Eq. (17) in Eq. (1) with the ansatz $f(R,T) = R + 2g(R)$ and varying the action with respect to the field ϕ, one obtains

$$P'(\phi)T + Q'(\phi) = 0 \ , \tag{18}$$

where the prime denotes the derivative with respect to the field ϕ. Here, it is considered that the action is unchanged under the variation of the auxiliary field. Then, the derivative of the function $g(T)$ can be written as

$$g_T(T) = P(\phi) + \left[P'(\phi)T + Q'(\phi)\right] \frac{\partial \phi}{\partial T} \ . \tag{19}$$

Making use of Eq. (18), one gets

$$g_T(T) = P(\phi) \ . \tag{20}$$

From Eq. (10), one obtains

$$P(\phi) = -\frac{1}{2} - \frac{\dot{H}}{\rho(1+\omega)} \ , \tag{21}$$

and from Eqs. (8), (17) and (20), one finds

$$Q(\phi) = 3H^2 - \rho - \rho\,(3-\omega)\,P(\phi) \ . \tag{22}$$

Note that, since h is always positive for guaranteeing an expanding universe, an analysis can be done on the basis of ω. Then, one can observe two cases: $\omega > -1$ and $\omega < -1$. The latter ($\omega < -1$) is particularly interesting because even if $g(T)$ vanishes, singularities may appear.

I. The first case: $\omega < -1$

2.2.1 Treating the case $a(t) = \bar{a}(t_s - t)^{-h}$

With this scale factor, the energy density can be written as

$$\rho(t) = \rho_0 \bar{a}^{-3(1+\omega)} \left(t_s - t\right)^{3h(1+\omega)} \quad . \tag{23}$$

Substituting Eq. (22) into Eq. (21), one obtains

$$P(\phi) = -\frac{1}{2} - \frac{h\bar{a}^{3(1+\omega)}}{\rho_0(1+\omega)} \left(t_s - t\right)^{-2-3h(1+\omega)} \quad . \tag{24}$$

Now, injecting Eqs. (23) and (24) into Eq. (22), we get

$$Q(\phi) = \left[3h^2 + \frac{h(3-\omega)}{1+\omega}\right] \left(t_s - t\right)^{-2} + \frac{1}{2}\left(1-\omega\right)\rho_0 \bar{a}^{-3(1+\omega)} \left(t_s - t\right)^{3h(1+\omega)} \quad . \tag{25}$$

Since one can redefine ϕ properly, we may choose it to be the time coordinate, $\phi = t$.

For $\omega < -1$, being $h > 0$, one has that $3h(1+\omega)$ is always negative. In such a situation, three conditions can be considered:

- $-2 < 3h(1+\omega) < 0$,
- $3h(1+\omega) = -2$,
- $3h(1+\omega) < -2$.

Let us now analyse each expression of the scale factor, finding the corresponding $g(T)$.

A The first condition: $-2 < 3h(1+\omega) < 0$

In this case, as the singularity time is approached ($t \to t_s$), one has

$$P(t) \sim -\frac{h\bar{a}^{3(1+\omega)}}{\rho_0(1+\omega)} \left(t_s - t\right)^{-2-3h(1+\omega)} \quad , \tag{26}$$

and

$$Q(t) \sim \left[3h^2 + \frac{h(3-\omega)}{1+\omega}\right] \left(t_s - t\right)^{-2} \quad . \tag{27}$$

On the other hand, we can write the trace of the energy-momentum tensor in terms of the energy density as

$$\begin{aligned} T &= \rho - 3p \\ &= \rho\left(1 - 3\omega\right) \end{aligned} \tag{28}$$

from which one has $\rho = T/(1-3\omega)$, and then

$$t_s - t = \bar{a}^{\frac{1}{h}} \left[\rho_0(1-3\omega)\right]^{-\frac{1}{3h(1+\omega)}} T^{\frac{1}{3h(1+\omega)}} \quad . \tag{29}$$

Now, substituting Eq. (29) into Eqs. (26) and (27), one obtains

$$P(t) \to \bar{P}(T) = -\frac{h(1-3\omega)\bar{a}^{-\frac{2}{h}}}{1+\omega} \left[\rho_0(1-3\omega)\right]^{\frac{2}{3h(1+\omega)}} T^{-\frac{2}{3h(1+\omega)}-1} \quad , \tag{30}$$

$$Q(t) \to \bar{Q}(T) = \bar{a}^{-\frac{2}{h}} \left[3h^2 + \frac{h(3-\omega)}{1+\omega}\right] \left[\rho_0(1-3\omega)\right]^{\frac{2}{3h(1+\omega)}} T^{-\frac{2}{3h(1+\omega)}} \quad . \tag{31}$$

Using Eqs. (17), (30) and (31), we get

$$g(T)=h\bar{a}^{-\frac{2}{h}}\,(3h+2)\,[\rho_0(1-3\omega)]^{\frac{2}{3h(1+\omega)}}\,T^{\frac{-2}{3h(1+\omega)}}\ . \tag{32}$$

Thus, when the condition $-2<3h(1+\omega)<0$ is satisfied, the big rip can appear with the model

$$f(R,T)=R+2h\bar{a}^{-\frac{2}{h}}\,(3h+2)\,[\rho_0(1-3\omega)]^{\frac{2}{3h(1+\omega)}}\,T^{\frac{-2}{3h(1+\omega)}}\ . \tag{33}$$

B The second condition: $3h(1+\omega)=-2$

In this case, as $t\to t_s$ (considering $t_s=t+\epsilon$, with $\epsilon<<1$), using Eq. (29), Eqs. (24) and (25) become

$$\begin{aligned} P(t)\to\bar{P}(T) &= -\frac{1}{2}+\frac{3h^2\bar{a}^{-\frac{2}{h}}}{2\rho_0}\ , & (34)\\ Q(t)\to\bar{Q}(T) &= \frac{1+3h}{2(1+2h)}\left[\frac{1}{3}-\frac{h^2\bar{a}^{-\frac{2}{h}}}{\rho_0}\right]T\ . & (35)\end{aligned}$$

Now, substituting Eqs. (34) and (35) into Eq. (17), $g(T)$ is found and the corresponding $f(R,T)$ model reads

$$f(R,T)=R+\frac{2+3h}{3(1+2h)}\left[\frac{3h^2\bar{a}^{-\frac{2}{h}}}{\rho_0}-1\right]T\ . \tag{36}$$

Hence, when the condition $3h(1+\omega)=-2$ is satisfied, the big rip can occur with the model of Eq. (36).

C The third condition: $3h(1+\omega)<-2$

In this case, as the singularity time is approached, using Eq. (29), Eqs. (24) and (25) become

$$\begin{aligned} P(t)\to\bar{P}(T) &= -\frac{1}{2}\ , & (37)\\ Q(t)\to &= \frac{1}{2}(1-\omega)\rho_0\bar{a}^{-3(1+\omega)}\,(t_s-t)^{3h(1+\omega)} & \\ \bar{Q}(T) &= \frac{1-\omega}{2(1-3\omega)}T\ . & (38)\end{aligned}$$

Hence, substituting Eqs. (37) and (38) into Eq. (17), $g(T)$ can be obtained and the corresponding $f(R,T)$ model reads

$$f(R,T)=R+\frac{2\omega}{1-3\omega}T\ . \tag{39}$$

This model also allows the occurrence of the big rip with h satisfying the condition $3h(1+\omega)<-2$.

2.2.2 Treating the case $a(t)=\bar{a}\exp\left[\frac{h(t_s-t)^{1-\alpha}}{\alpha-1}\right]$

In this case, the appearance of both the big rip ($\alpha>1$) and type III singularity ($0<\alpha<1$) is possible. The expressions (21) and (22), in this case, read

$$\begin{aligned} P(t) &= -\frac{1}{2}-\frac{\alpha h\bar{a}^{3(1+\omega)}\,(t_s-t)^{-\alpha-1}}{\rho_0(1+\omega)}\exp\left[\frac{3h(1+\omega)(t_s-t)^{1-\alpha}}{\alpha-1}\right]\ , & (40)\\ Q(t) &= 3h^2\,(t_s-t)^{-2\alpha}+\frac{\alpha h(3-\omega)(t_s-t)^{-\alpha-1}}{1+\omega} & \\ &+ \frac{1}{2}\,(1-\omega)\,\rho_0\bar{a}^{-3(1+\omega)}\exp\left[\frac{-3h(1+\omega)(t_s-t)^{1-\alpha}}{\alpha-1}\right]\ . & (41)\end{aligned}$$

Making use of Eq. (28), one gets

$$t_s - t = \ln^{\frac{1}{1-\alpha}}\left[[\rho_0(1-3\omega)]^{\frac{\alpha-1}{3h(1+\omega)}}\,\bar{a}^{\frac{1-\alpha}{h}}\,T^{\frac{1-\alpha}{3h(1+\omega)}}\right] . \tag{42}$$

- Type III singularity, $0<\alpha<1$

As the singularity time is approached, we get

$$P(t) = \frac{-\alpha h \bar{a}^{3(1+\omega)}(t_s-t)^{-\alpha-1}}{\rho_0(1+\omega)} , \tag{43}$$

$$Q(t) = \frac{\alpha h(3-\omega)\,(t_s-t)^{-\alpha-1}}{1+\omega} . \tag{44}$$

Substituting Eq. (42) into Eqs. (43) and (44), and using Eq. (17), one can obtain $g(T)$, and the $f(R,T)$ model that leads to the type III singularity reads

$$f(R,T) = R + \frac{2\alpha h\bar{a}^{3(1+\omega)}}{\rho_0(1+\omega)}\left[\frac{\rho_0(3-\omega)}{\bar{a}^{3(1+\omega)}} - T\right]\ln^{\frac{\alpha+1}{\alpha-1}}\left[[\rho_0(1-3\omega)]^{\frac{\alpha-1}{3h(1+\omega)}}\,\bar{a}^{\frac{1-\alpha}{h}}\,T^{\frac{1-\alpha}{3h(1+\omega)}}\right] . \tag{45}$$

- Big rip, $\alpha>1$

In this case, as the singularity time is approached, Eqs. (40) and (41) become

$$P(t) \sim -\frac{\alpha h\bar{a}^{3(1+\omega)}(t_s-t)^{-\alpha-1}}{\rho_0(1+\omega)} , \tag{46}$$

$$Q(t) \sim \frac{1}{2}(1-\omega)\,\rho_0\bar{a}^{-3(1+\omega)}\exp\left[\frac{-3h(1+\omega)(t_s-t)^{1-\alpha}}{\alpha-1}\right] . \tag{47}$$

On the other hand, from Eq. (42) we can write

$$\exp\left[\frac{-3h(1+\omega)(t_s-t)^{1-\alpha}}{\alpha-1}\right] = \frac{\bar{a}^{3(1+\omega)}\,T}{\rho_0(1-3\omega)} , \tag{48}$$

from which, we get

$$P(t)\to\bar{P}(T) = -\frac{\alpha h\bar{a}^{3(1+\omega)}}{\rho_0(1+\omega)}\ln^{\frac{\alpha+1}{\alpha}-1}\left[[\rho_0(1-3\omega)]^{\frac{\alpha-1}{3h(1+\omega)}}\,\bar{a}^{\frac{1-\alpha}{h}}\,T^{\frac{1-\alpha}{3h(1+\omega)}}\right] , \tag{49}$$

$$Q(t)\to\bar{Q}(T) = \frac{1-\omega}{2(1-3\omega)}\,T , \tag{50}$$

and $g(T)$ can be obtained through Eq. (17), leading to the model

$$f(R,T) = R + \left\{\frac{1-\omega}{1-3\omega} - \frac{2\alpha h\bar{a}^{3(1+\omega)}}{\rho_0(1+\omega)}\ln^{\frac{\alpha+1}{\alpha}-1}\left[[\rho_0(1-3\omega)]^{\frac{\alpha-1}{3h(1+\omega)}}\,\bar{a}^{\frac{1-\alpha}{h}}\,T^{\frac{1-\alpha}{3h(1+\omega)}}\right]\right\}T , \tag{51}$$

which can allow the occurrence of the big rip.

2.2.3 Treating the case $a(t) = \bar{a}\exp\left\{-(t_s-t)\left[H_0 - \frac{h(t_s-t)^{-\alpha}}{\alpha-1}\right]\right\}$

Here, the expressions (21) and (22) take new forms:

$$P(t) = -\frac{1}{2} - \frac{\alpha h\bar{a}^{3(1+\omega)}(t_s-t)^{-\alpha-1}}{\rho_0(1+\omega)}\exp\left\{-3(1+\omega)(t_s-t)\left[H_s - \frac{h(t_s-t)^{-\alpha}}{\alpha-1}\right]\right\} , \tag{52}$$

$$\begin{aligned} Q(t) &= 3\left[h(t_s-t)^{-\alpha}+H_s\right]^2 + \frac{\alpha h(3-\omega)(t_s-t)^{-\alpha-1}}{1+\omega} \\ &+ \frac{1}{2}\rho_0\bar{a}^{-3(1+\omega)}\exp\left\{3(1+\omega)(t_s-t)\left[H_s - \frac{h(t_s-t)^{-\alpha}}{\alpha-1}\right]\right\} . \end{aligned} \tag{53}$$

- Sudden singularity , $-1 < \alpha < 0$.

As the singularity time is approached, Eqs. (52) and (53) can be written as

$$P(t) = -\frac{\alpha h \bar{a}^{3(1+\omega)}(t_s-t)^{-\alpha-1}}{\rho_0(1+\omega)} e^{-3H_s(1+\omega)(t_s-t)} \ , \tag{54}$$

$$Q(t) = \frac{\alpha h(3-\omega)(t_s-t)^{-1-\alpha}}{1+\omega} \ . \tag{55}$$

On the other hand, the trace can be put in the following form:

$$T = \rho_0(1-3\omega)\bar{a}^{-3(1+\omega)} \exp\left\{3(1+\omega)(t_s-t)\left[H_s - \frac{h(t_s-t)^{-\alpha}}{\alpha-1}\right]\right\} \ , \tag{56}$$

which, near the singularity, leads to

$$t_s - t = \ln\left[[\rho_0(1-3\omega)]^{-\frac{1}{3H_s(1+\omega)}} \bar{a}^{\frac{1}{H_s}} T^{\frac{1}{3H_s(1+\omega)}}\right] \ . \tag{57}$$

Substituting Eq. (57) into Eqs. (54) and (55), and using Eq. (17), $g(T)$ is obtained and the model corresponding to the sudden singularity reads

$$f(R,T) = R + 4\alpha h \ln^{-\alpha-1}\left[[\rho_0(1-3\omega)]^{-\frac{1}{3H_s(1+\omega)}} \bar{a}^{\frac{1}{H_s}} T^{\frac{1}{3H_s(1+\omega)}}\right] \ . \tag{58}$$

- Type IV singularity , $\alpha < -1$.

In this case, close to the singularity, Eqs. (52) and (53) become

$$P(t) \sim -\frac{1}{2} \ , \tag{59}$$

$$Q(t) \sim 3H_s^2 + \frac{1}{2}(1-\omega)\rho_0\bar{a}^{-3(1+\omega)} \ . \tag{60}$$

Here, as the singularity time is approached, the trace of the energy-momentum tensor takes on the simple expression

$$T = \rho_0\bar{a}^{-3(1+\omega)}(1-3\omega) \ , \tag{61}$$

from which, using Eq. (17), $g(T)$ is easily obtained and then, the $f(R,T)$ models which allows the occurrence of the type IV singularity reads

$$f(R,T) = R + \frac{2\omega}{1-3\omega}T + 6H_s^2 \ . \tag{62}$$

II. Commenting the case $\omega > -1$

• Considering the scale factor of Eq. (14)

In this case, the quantity $3h(1+\omega)$ is always positive, such that $-2-3h(1+\omega) < 0$. This leads to the same expressions as that in Eqs. (26) and (27) for $P(t)$ and $Q(t)$, respectively. Then, the corresponding

$f(R,T)$ model is already given in Eq. (33). Therefore, the big rip can also appear, even if $\omega > -1$. It is important to note that if $g(T)$ vanishes, the fluid corresponds to quintessence and then, the type III cannot occur. Our result is clearly the effect of the presence of a part depending on the trace of the energy-momentum tensor in the gravitational action.

- **Considering the scale factor of Eq. (15)**

In this case, for $0 < \alpha < 1$, corresponding to the appearance of the type III singularity, the expressions for $P(t)$ and $Q(t)$ are the same as in Eqs. (43) and (44) and then the corresponding $f(R,T)$ model is the one in Eq. (45).

For the case $\alpha > 1$, corresponding to the appearance of the big rip, the situation is quite different. As the singularity time is approached, the expressions (40) and (41) become Eqs. (54) and (55), respectively, and the corresponding $f(R,T)$ model is the same as in Eq. (58).

- **Considering the scale factor of Eq. (16)**

For $-1 < \alpha < 0$, the case in which the sudden singularity may appear, Eqs. (52) and (53) take the form (54) and (55) and consequently, the corresponding model is given in Eq. (58).

For $\alpha < -1$, the case in which the type IV singularity may appear, the expressions (52) and (53) become (59) and (60), respectively, and the corresponding model is given in Eq. (62).

3 Quantum effects near finite time singularity

We may check the avoidance of finite time singularities by taking into account quantum effects. In this work we address conformal anomaly. Near the future finite-time singularity ($t \to t_s$) the curvature diverges. Due to their dependence on the curvature, quantum effects coming from conformal anomaly also become important. In such a situation, all classical considerations have to be revised and any claim about the appearance of future finite-time singularity cannot be justified without an account of quantum effects. One may incorporate the massless quantum effects by taking into account the conformal anomaly contribution as a backreaction near the singularity. The conformal anomaly T_A has the following well-known expression [32]

$$T_A = b\left(F + \frac{2}{3}\Box R\right) + b'G + b''\Box R \ , \tag{63}$$

where F is the square of the four-dimensional Weyl tensor and G is Gauss-Bonnet invariant, given by

$$F = \frac{1}{3}R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \ , \tag{64}$$

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \ . \tag{65}$$

Explicitly, if there are N scalars, $N_{1/2}$ spinors, N_1 vector fields, N_2 gravitons and N_{HD} higher derivative conformal scalars, b and b' are expressed as

$$b = \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{HD}}{1920\pi^2} , \tag{66}$$

$$b' = \frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{HD}}{5760\pi^2} , \tag{67}$$

whereas b'' is an arbitrary constant whose value can be shifted by the finite renormalization of the local counter-term R^2.

Now, by taking into account quantum contribution from the conformal anomaly, Eq. (7) is modified and its trace can be written as

$$-R = [1 + 2g_T(T)] + 8pg_T(T) + 4g(T) + T_A \quad . \tag{68}$$

From now on, we assume that the ordinary fluid is pressureless, i.e. $p = 0$. Then, Eq. (68) becomes

$$-R = [1 + 2g_\rho(\rho)] + 4g(\rho) + T_A \quad . \tag{69}$$

For the FLRW universe, it is clear that

$$F = 0 , \quad G = 24\left(\dot{H}H^2 + H^4\right) \quad . \tag{70}$$

Let us focus our attention on the case $2b + 3b'' = 0$ and set $b' = 1$. Then, one gets $T_A = G$. We may analyse quantum effects coming from conformal anomaly near each type of singularity.

3.1 Conformal anomaly near the big rip

In this case, one can observe two conditions on the parameter α, that is, $\alpha = 1$ and $\alpha > 1$.

• The case $\alpha = 1$

Here, the curvature behaves as $R \propto (t_s - t)^{-2}$, the trace as $T \propto (t_s - t)^{3h(1+\omega)}$, and $T_A \propto (t_s - t)^{-4}$.

Here, we may distinguish the three conditions previously mentioned; $-2 < 3h(1+\omega) < 0$, $3h(1+\omega) = -2$ and $3h(1+\omega) < -2$.

For $-2 < 3h(1+\omega) < 0$, the corresponding expression of $g(T)$ is Eq. (32), and then, $g(T) \propto Tg_T(T) \propto (t_s - t)^{-2}$. It appears that, as the singularity time is approached, the conformal anomaly term dominates other others terms. Hence, the big rip may be avoided.

For $3h(1+\omega) = -2$, the corresponding expression for $g(T)$ is that coming from Eq. (36), and then $g(T) \propto Tg_T(T) \propto (t_s - t)^{-2}$. Also, in this case, the conformal anomaly term dominates and the big rip may also be avoided.

For $3h(1+\omega) < -2$, the corresponding expression of $g(T)$ is that coming from Eq. (39), and $g(T) \propto Tg_T(T) \propto (t_s - t)^{3h(1+\omega)}$. Here, it can be observed that, even if the conformal anomaly is not taken into

account the big rip may be avoided from the contribution of the matter in the gravitational part of the action. This can be viewed as a reason for taking into account the trace T term in the gravitational part since, in this case, the big rip can be avoided without the need of quantum effect.

- The case $\alpha > 1$

Here, as the singularity time is approached, the curvature behaves as $R \propto (t_s - t)^{-2\alpha}$, the conformal anomaly as $(t_s - t)^{-4\alpha}$, and the ordinary trace as $T \propto e^{(t_s-t)^{3h(1+\omega)}}$. The corresponding expression of $g(T)$ is that coming from Eq. (51) and one gets, $g(T) \propto Tg_T(T) \propto (t_s - t)^{\frac{1-\alpha}{\alpha}} e^{(t_s-t)^{3h(1+\omega)}}$, that one can assimilate to $(t_s - t)^{-2N}$, where $N = N(\alpha, \omega)$ is a positive function of α and ω. This assumption is possible since, for $\alpha > 1$, $\frac{1-\alpha}{\alpha} < 0$, and also, as we are dealing with the case $\omega < -1$, $3h(1 + \omega) < 0$. It is clear here that we also have $-4\alpha < -2\alpha$, and then, the conformal anomaly term dominates the curvature one. One can conclude that for $\alpha > 1$, quantum effects from conformal anomaly may allow the avoidance of the big rip. Note also that, if for some values of $\alpha > 1$ and $\omega < -1$, $N > 2$, then, we regain the situation on which $f(R, T)$ itself lead to the avoidance of the big rip, from the ordinary trace term in the gravitational part of the action, without the need of quantum effects.

3.2 Conformal anomaly near the sudden singularity

In this case, $-1 < \alpha < 0$, and as the singularity is approached, the curvature behaves as $R \propto (t_s - t)^{-\alpha-1}$, the ordinary trace as $T \propto e^{(t_s-t)^{3H_s(1+\omega)}}$ and the conformal anomaly as $T_A \propto (t_s - t)^{-3\alpha-1}$. The corresponding expression of $g(T)$ is that coming from Eq. (58). Here, one gets $g(T) \propto Tg_T(T) \propto (t_s - t)^{-\alpha-1}$. In this case, it appears that the avoidance of the sudden singularity must come from the quantum effect. In this way, one can observe that for $-1 < \alpha < -1/3$, $-3\alpha-1 > 0$ while $-\alpha-1 < 0$. Then, the sudden singularity is robust again quantum effect coming from conformal anomaly. For $-1/3 < \alpha < 0$, $-\alpha - 1 < -3\alpha - 1 < 0$. Also in this situation, the sudden singularity cannot be avoided from conformal anomaly contribution.

3.3 Conformal anomaly near the singularity of type III

This type of singularity occurs for $0 < \alpha < 1$, and around the singularity, the curvature behaves as $R \propto (t_s-t)^{-\alpha-1}$, the ordinary trace as $T \propto e^{(t_s-t)^{3h(1+\omega)}}$ and the conformal anomaly as $T_A \propto (t_s-t)^{-3\alpha-1}$. The corresponding expression of $g(T)$ is that coming from Eq. (45), and we gets $g(T) \propto Tg_T(T) \propto e^{(t_s-t)^{3h(1+\omega)}}$. As $0 < \alpha < 1$, one gets $-3\alpha - 1 < -\alpha - 1 < 0$. Then, the singularity of type III may be avoided from conformal anomaly effects.

3.4 Conformal anomaly near the singularity of type IV

The condition of occurrence of this type of singularity is $\alpha < -1$, and this case the curvature is not singular, but form it first to higher derivatives diverge as the singularity time is approached. The conformal anomaly term behaves as $(t_s - t)^{-3\alpha-1}$. Note that in this case, we always get $-3\alpha - 1 > 0$. In such a situation the singularity is always robust against the conformal anomaly and cannot be avoided.

Another important point to be observed is the case where $\omega > -1$. The special aspect in this case is that the quantity $3h(1+\omega)$ is always positive, and then, any singular behaviour cannot be observed from the ordinary trace contribution. Hence, the unique possibility for avoiding the singularities must come from quantum effects.

4 Conclusion

We considered the modified $f(R,T)$ theory of gravity where R is the curvature scalar and T the trace of the energy-momentum tensor. We focused our attention in reconstructing the models of this theory, that may lead to finite-time future singularities. We assumed a suitable expression for the Hubble parameter in order to provide the expansion of the universe and realise the four finite-time future singularities, by adjusting an input parameter α. A special expression $f(R,T) = R+2g(T)$ is assumed, viewed as a T-term correction to the Einstein-Hilbert term, R. From this, differential equation for $g(T)$ are obtained through the equations of motion, for some values of the parameter α. In each case, the differential equation is solved leading to the $f(R,T)$ model that leads to finite-time future singularity.

As the curvature diverges at singularity time, quantum effect may be introduced. In this way, conformal anomaly is introduced and its effects at singularity time in analysed. We observed that conformal anomaly cannot remove the sudden singularity nor the type IV singularity. However, for some values of the parameter α, the big rip and the type III singularity may be avoided. Another important result that we find is that, even without the contribution of quantum effect, the big rip and the type III singularity may be avoided from the contribution of the ordinary trace terms, for some values of the parameter α. This is a reason for considering $f(R,T)$ in the study of some scenarios of evolution of the universe.

In the case of type II (sudden singularities) and type IV singularities, we observed that neither conformal anomaly or the terms coming from the classical trace allow the avoidance of these singularities. Still, in order to check the possible avoidance of type II and IV singularities, the viscosity of the fluid characterising the matter content may be introduced. We leave this as a future work.

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